# SAR Images Co-registration Based on Phase Congruency Algorithm

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Abstract— As the first step in image processing operations, corner detection is a vital procedure where it can be applied to many applications such as matching features, image registration, image mosaicking, change detection.... U.S. Registration of images can be described as the process of getting the pixel location misaligned between two or more images. A modified corner detector is proposed in this paper based on a combination of both phase congruence, later named PC, and Harris corner detector where PC image can provide fundamental and meaningful features despite complex changes in intensity. The performance was similar to detectors for the Shi-Tomasi, FAST, and Harris corner. Experiments are carried out using simulated images. As metric metrics, MSE (mean square error) and PSNR (peak signal-to-noise ratio) are used. The experimental results verify the effectiveness where the advantages of image constitutional representation are utilized, allowing the extraction of the powerful key points since it preserves the robustness of the co-registration process using image frequency properties that are variant to illumination. It also has the ability to produce reasonable results as opposed to state-of - the-art approaches such as Shi-Tomasi, FAST, and Harris algorithms.-

*Keywords*- Feature Detection, Harris Corner Detector, Image Registration, Phase Congruency, Random Sample Consensus (RANSAC).

# I. INTRODUCTION

Performing the process of image registration allows finding the correspondence between the input images by estimating the necessary transformation matrix to adjust the positions of a selected point in the sensed image with its corresponding one in the matched image. The detection of features is an essential step in interpreting the images where they can be used as image registration in various applications. Registration of images has two main approaches; the first is called the areabased matching method; denoted ABM; where the presence of the feature is guaranteed rather than its identification. The other approach is called the feature-based matching method; denoted FBM; where the function detection mechanism plays a vital step. Generally speaking, each approach has advantages that differentiate it from the other and at the same time; it has its drawbacks that could be recurred with the other approach

[1-5]. Features may be global or local, and each has its own features [6-10].

The use of local features is preferred since it is possible to achieve the task of defining a group of noteworthy points, with unique properties [6-11]. Multiple feature detection techniques were developed considering the required application where their performance behavior is related to design parameters. So; the Registration process does not have a common ideal algorithm. Each detector should have certain features and also a good feature [2, 8-11].

Because data imperfections affect the detection accuracy, the detection process target collapses the similar characteristics given these imperfections. In the case of information shape stability structure, issues related to translation; rotation; and scaling can be easily treated where w.r.t projective and affine problems are more difficult to handle in the detection process [8, 9, 12, 13]. So; feature detection methodology should not be sensitive to the distortions in the image. There have been many suggested techniques where the categorization of feature detectors can be accomplished taking into account operating concept and scale [1-4, 8-13].

Generally speaking; the goal of the registration is the ability to have the right transformation matrix to represent points in one image to its congruence in the other, depending primarily on the detection of features to align images that may be two or more. The values of matrix parameters should satisfy the required similarity metric estimates, resulting in the required proximity or suitability stage between images being obtained by increasing the mutual input information. Furthermore, the algorithm of one registration could not be used as a generic approach for all images as each algorithm is designed to perform a specific mission and may fail to perform another. Registration algorithms consist of three phases: (1) detection of key points; (2) possession of the required geometric transformation model linking these key points; and (3) cost function for similarity metric measurements providing the degree of suitability between inputs. Many detectors approaches; based on feature detection; use the gray information behavior to extract key points, so it's easily influenced, by contrast, noise, and lighting variations. These variations cause unstable and imprecise localization of the features. Some versions of these detectors were created later [1-12].

## II. HARRIS CORNER DETECTOR

Harris detector [14] is a uni-scale detector based on intensity derivatives to handle Moravec's detector limitations [15] by acquiring intensity variations for all different orientations. Its use is primarily for gray scale pictures, where it can be counted as a mishmash of edge and corner detectors [8-14]. The main idea is to estimate own values for regions with small patches, then use the greatest ind ividual values to specify a predefined function combined with a threshold to examine the corners as shown in Figure 1(a) [8-10, 13].

The matrix of covariance with derivatives is used to detect the position of the fastest and slowest variations for feature orient ation

[8, 10]. Pixel gradients (Ix, Iy) in x and y directions are calcul ated for all pixels conducting a correlation matrix (M) [8, 11-14], respectively:

$$M = \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \tag{1}$$

All matrix's components are smoothed improving detector's robustness. Combining the estimations of eigenvalues with a certain threshold value depicts a feature to be a corner point or not. This is known as pixel score  $(R_s)$  which can be determined as following [8-14]:

$$R_{s} = \lambda_{1}\lambda_{2} - k(\lambda_{1} + \lambda_{2})^{2} \tag{2}$$

where  $\lambda_1$  and  $\lambda_2$  are eigenvalues of (M) and k is the detector sensibility agent, in general it is within range 0.04:0.06. The pixel is considered to be a corner if both eigenvalues  $\lambda_1$  and  $\lambda_2$  are high as shown in figure 1(a). The computation cost of eigenvalues estimation is high, so that an enhancement will be applied using trace and determinant of matrix M providing oncoming relationship to estimate pixel's score  $R_s$  as following [8-10, 13, 14].

$$R_{s} = \det(\mathbf{M}) - k(trace(M))^{2} \tag{3}$$

Enhancements over harris detector was proposed allowing utilization for color data [2, 6-10, 13]. Shi and Tomasi, eigenvalues detector, [16] developed an optimization approach based on harris method permitting the utilization of minimum eigenvalues for differentiating key points and hence controls and facilitates the cost computations of harris detector. When  $R_{\rm s}$  is higher than pre-determined threshold value, key point is a corner as shown in figure 1(b) where the pixel's score is estimated as following [8-10, 16].

$$R_{s} = \min(\lambda_{1}, \lambda_{2}) \tag{4}$$

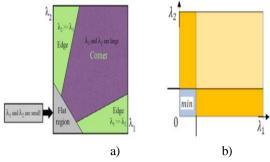


Figure 1. Idea of both a) harris and b) Shi, Tomasi corner detectors.

Harris detector has various advantages as invariant to rotation; illumination; and changes in translation, high stability, high repeatability of rotational invariants, high informativity, accurately localized, less criteria for computations. Given all of these, harris detector has drawbacks such as noise sensitivity, cost computations, and variance to high-scale variations [8-11, 13]. It was proposed that Harris Laplace and Harris Affine act as an affine invariant corner detector designed for multi-scale operations, depending on the harris method. It earns the property of robustness to transformations from types of perspective as it is invariant to affine them and offers a representative set of points with highly repeatable dimensions and insensitivity to scale; illumination; rotation; and variations in noise. The drawbacks of use are that it offers reduced redundant key points and is not effective for affine problems [8-14].

## III. PHASE CONGRUENCY (PC)

Much of the approaches to pre-processing images rely primarily on the power of correct detection, localization, and matching of features. Solving the problem of correct detection and location requires a robust tool that has a high invariance to lighten and scale. PC acts as a low-level operation for the detection of features to be applied to the operational levels. Having an invariant feature, along with wide ranges under different image-capture conditions, is a vital step in providing a robust material for processing operations [17-20]. Deciding to consider a function as a key point depends on the threshold value that is considered [1-11, 20, 21].

PC approach uses the information given by image representation in the frequency domain. This is the greatest benefit for PC approach where the function is calculated at a point where the overall phase Fourier elements, taking into account local energy calculations E, are proportional to PC measurements. The PC values are in the range from 0 to 1 where the value 0 shows no congruency and the value 1 shows outstanding congruency. This dimensional measure allows the selection of preceding image construction features depending on the predetermined threshold value. PC has limits on itself as[ 17-27]:1) it is highly affected by noise since it has a normalized amount from 0 to 1;

2) it depends mainly on the fourier components which may be very small or only one component exists, depending on the fourier component's amplitude  $A_n$  and function of local energy

E(x) at the investigated feature (x). So, frequencies expansion plays an important role.

$$\sum A_n \; ; \; E(x) \; ; \; 0 \tag{5}$$

$$\sum_{n} A_{n} ; E(x); 0$$

$$\sum_{n} A_{n} = E(x)$$
(6)

## 3) Not providing accurate localization for the feature.

A local energy model assumes that the feature is drawn up at a certain point where Fourier components have a maximum phase such that PC calculations at any angle provide a distinctly key point. Figure 2 shows how to make triangular and square waveforms using Fourier-chain components. Figure 3 illustrates an example of a step function interpolation to a line and its grating profiles using the spread chain s(x) where  $\pi$  is a displacement for the occurrence of PC and varies from (n/2)0 to [17-20].

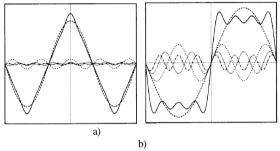


Figure 2. Making up of (a) triangular and (b) square waveforms

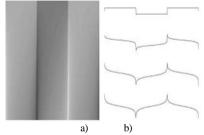


Figure 3. Step feature interpolation to a line (left) and its grating profiles (right)

W.r.t a signal I(x), 1-D, the function of PC related to the expansion of fourier, at feature x will be estimated as where  $A_n$  is the n<sup>th</sup> fourier component's amplitude,  $\phi_n(x)$  is the fourier component's phase at location x, and  $\overline{\phi}(x)$  is the amount of weighted mean local phase maximizing [17, 18, 21, 28].

$$s(x) = \sum_{n=0}^{\infty} \frac{1}{(2n+1)} \sin[(2n+1)x + \phi]$$
 (7)

$$PC(x) = \max_{\overline{\phi}(x) \in [0,2\pi]} \frac{\sum_{n} A_n \cos(\phi_n(x) - \overline{\phi}(x))}{\sum_{n} A_n}$$
(8)

Points having maximal PC can be estimated by direct looking for the local energy peak position at point x [17-20]

$$E(x) = \sqrt{F^{2}(x) + H^{2}(x)}$$
 (9)

where F(x) is the signal I(x) without DC composition, and H(x) is its Hilbert transform. Then; E(x) can be estimated as PC scaling [17-20, 29, 30]

$$E(x) = PC(x) \sum A_n \tag{10}$$

So, the function of local energy E, dimensional amount, is direct proportion to the function's measurements of PC as mentioned before. Before normalization process, the noise will be estimated and then subtracted from E(x). This process reduces the noise response and PC will be modified as [17-27, 30, 311

$$PC(x) = \frac{\lfloor E(x) - T \rfloor}{\sum A_n(x) + \varepsilon}$$
 (11)

where the proper account of  $(\varepsilon)$  relies on accuracy of ability of information extraction and used to avoid dividing by zero. The ringed amount |E(x)-T| equals to itself at the moment having a positive value, and elsewhere equals zero. So, a calculation of trust that the point at location x is considerably related to the noise level [17-20,21, 31].

The frequency spread acts as an important parameter where PC point is considerable when occurring along an extended frequencies range. Since smoothing process minimizes the high components of frequency causing reduction of expansion of frequency, so some modifications, weight function W(x), should be added to weight the PC by counting the aggregate of the magnitudes of frequency components and divide by the largest frequency component. So, the spread function s(x) will be as [17-21, 30-32]

$$s(x) = \frac{1}{N} \left( \frac{\sum_{n} A_{n}(x)}{A_{\max}(x) + \varepsilon} \right)$$
 (12)

where N is overall number of scales and  $A_{max}(x)$  is largest frequency component at (x), thus [17-21, 30-32]

$$PC(x) = \frac{W(x) \lfloor E(x) - T \rfloor}{\sum A_n(x) + \varepsilon}$$
(13)

The problem of PC localization is still existing after the modifications of spread function. That is because there is a proportional relation between the local energy E(x) and divergence of phase angle  $\phi_n(x)$  from total average of phase angle  $\overline{\phi}(x)$  will given as

$$\Delta\Phi(x) = \cos(\phi_n(x) - \overline{\phi}(x)) - \left|\sin(\phi_n(x) - \overline{\phi}(x))\right| \quad (14)$$

where  $\Delta\Phi(x)$  is the divergence of phase, thus

$$PC(x) = \frac{\sum_{n} W(x) \left[ A_{n}(x) \Delta \Phi_{n}(x) - T \right]}{\sum_{n} A_{n}(x) + \varepsilon}$$
(15)

where T is the estimated effect of noise [17-22, 26, 30-33].

To extend the PC algorithm to 2D images, a thorough studies of the conduct of local energy (E) w.r.t 2D image were discussed. Applying the PC approach for 2D images requires applying it for each direction separately and finally combining the results. The final PC equation at location (x) will given as [17-33].

$$PC(x) = \frac{\sum_{o} \sum_{n} W_{o}(x) \left[ A_{no}(x) \Delta \Phi_{no}(x) - T_{o} \right]}{\sum_{o} \sum_{n} A_{no}(x) + \varepsilon}$$
(16)

where (o) represents the orientation index where the process of normalizing the energy to construct PC is carried out after combining overall local energies for all possible orientations. Also, the compensation of the noise is achieved independently for each orientation [17-23, 29-33]. In order to implement the PC algorithm, the required procedures start by applying quadrature pairs of wavelets filters over number of orientations, equals 6, and number of different scales, equals 4, for each point in the image. This process will be performed for x and y directions independently providing the frequency data. Then, the PC covariance matrix will be calculated as

$$G = \begin{bmatrix} \sum PC_x^2 & \sum PC_x PC_y \\ \sum PC_x PC_y & \sum PC_y^2 \end{bmatrix}$$
 (17)

where  $PC_x$  and  $PC_y$  are PC components in x and y directions. The Summation is to acquire the values for all orientations [17-22]. Having large values of minimum and maximum PC moments indicate that the point has a powerful being a corner for minimum PC moments and prominence of point surely to be edge for maximum PC moments [17-21, 24, 30-33].

# IV. METHODOLOGY

The illustrated method of registration is a mix of ABM and FBM techniques. It will be carried out starting by cross correlation method as ABM technique, coarse registration, and then applying the result to the proposed corner detector to perform the fine registration process where processing of PC images using the harris corner detector to get the corner points. Second; using Random Sample Consensus (RANSAC) algorithm [34-36] to eliminate the undesired corner points in order to have the correct transformation matrix. Finally; the sensed image will be interpolated.

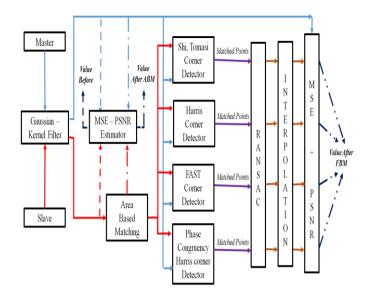


Figure 4. General workflow of the proposed registration method

## A. Proposed Framework

Figure 4 illustrates the proposed framework. After getting slave and master images, shift between input images; PSNR; and MSE are estimated before initiating process of registration to find out the performance of the proposed approach. As mentioned before, the depicted method is mainly dependent on a combination between ABM and FBM techniques.

First; ABM technique, represented in correlation method, will be applied performing the stage of coarse registration. A shifted slave image will be generated and estimations related to the shift between input images, PSNR, and MSE, after coarse registration, will be calculated.

Second; FBM technique, represented in the proposed corner detector, will be carried out acting as the stage of fine registration. The detector will generate a number of matched points in both master and shifted slave images. These points are classified as inliers (desired) and outliers (undesired). The extracted matched points will be applied to RANSAC algorithm to generate a geometric transformation matrix satisfying the cost function of RANSAC, can be considered as an optimization approach, to eliminate the outliers points which will affect the performance of generation of transformation matrix and only pick out the inliers points which are most correctly matched points between the input images to increase the fineness of the desired matrix of transformation performing better fine registration as shown in figure 5 [1-13, 34-36].

Finally, the matrix of transformation is required to interpolate the shifted slave image produced from ABM method. Also; after fine registration, calculations related to the shift between input images, PSNR, and MSE will be estimated. Widespread utilized detectors (Harris [14], Eigenvalues [16], and Features from Accelerated Segment Test (FAST) [37]) are used to estimate and evaluate the

# B. Experimental Dataset, Results, and Analysis

Two pairs of simulated images are used. Images having different sizes as presented in table I. Specifications of detectors related to each image are presented in table II. The demonstrated data allow to examine simulated images accompanied by approximately small and little high shift. Results show that the illustrated method provides a reasonable output emulated to the ordinarily used detectors (Minimum Eigenvalues, FAST, and Harris).

After coarse registration process, the value of shift between the inputs equals (0,0), but a sub pixel shift exists and not sensed. After fine registration process, the value of shift between the inputs equals (0,0) exactly. Table III and table IV show the experimental results where performance of depicted approach is compared to the ordinarily used detectors as mentioned above.

TABLE I. TABLE I USED DATASET

Specification	Specifications of simulated images			
Pair No.	(1) (2)			
Image Type	Simulated			
Sensor	-			
Pixel Shift	0, 1	199, 6		
Size	1502*1148	9000*2500		

TABLE II. TABLE II SPECIFICATIONS OF DETECTORS

Specifications of used detectors					
Harris threshold	1.0e <sup>-08</sup>	1.0e <sup>-06</sup>			
Eigenvalues threshold	1.0e <sup>-06</sup>	1.0e <sup>-06</sup>			
FAST threshold	5	1200			
PROPOSED threshold	1.0e <sup>-08</sup>	1.0e <sup>-08</sup>			

TABLE III. TABLE III EXPERIMENTAL RESULTS OF IMAGE PAIR NO. 1

		Pair No. (1), Simulated Images				
		Harris	FAST	Eigenvalues	PROPOSED	
ē	Peak			0, 1		
Before	MSE			1.8154		
ABM	PSNR	45.5410				
	Peak	0, 0				
	MSE	1.1243				
	PSNR	47.6222				
	Number	Fail	Fail	70	36	
FMB	Peak	-	-	0,0	0,0	
	Peak MSE	-	-	1.1241	1.1243	
	PSNR	-	-	47.623	47.6222	

TABLE IV. TABLE IV EXPERIMENTAL RESULTS OF IMAGE PAIR No. 2

-					
		Pair No. (2), Simulated Images			
		Harris	FAST	Eigenvalues	PROPOSED
Before	Peak	199, 6			
	MSE			1.8465	
	PSNR	45.4674			
ABM	Peak	0, 0			
	MSE	1.3164			
	PSNR	46.9369			
	Number	46	9	51	20
FMB	Peak	0, 0	0,0	0,0	0, 0
	MSE	1.3159	1.3163	1.3161	1.3164
	PSNR	46.9385	46.9372	46.9379	46.9370

#### V. CONCLUSION

A proposed corner detector is introduced using harris corner concept based on PC algorithm. Proposed corner detector is a highly localized tool for feature identification (corners or edges) since it utilizes both phase congruency (PC) algorithm and harris corner concept where PC reserves an inequality invariant manner by concatenating information of PC over different orientations. It enables detecting of features independent on illumination changes.

Also, PC map for corner or edges can be generated using minimum and maximum moments of the PC covariance matrix. The experimental results verify the effectiveness of the proposed approach where it utilizes the privileges of image constitutional depicting, allowing extraction of the most powerful key points since it preserves robustness of coregistration process using the image frequency properties which are not variant to illumination. Also, it has the ability to give reasonable results compared to the state of art method as Shi-Tomasi, FAST, and Harris algorithms.

## REFERENCES

- [1] Mahmoud Hassaballah, Khalid M. Hosny, "Analysis and Evaluation of Keypoint Descriptors for Image Matching", in "Recent Advances in Computer Vision Theories and Applications," vol. 804, Springer Nature Switzerland, 2019, pp. 113-140.
- [2] Ertugrul Bayraktar, Pinar Boyraz, "Analysis of feature detector and descriptor combinations with a localization experiment for various performance metrics", *Turkish Journal of Electrical Engineering & Computer Sciences*, vol. 25, pp. 2444-2454, May, 2017. Accessed on: Sep., 20, 2016, DOI: 10.3906/elk-1602-225, [Online].
- [3] Abdelhameed S. Eltanany, M. S. Elwan, and A. S. Amein, "Key Point Detection Techniques", *Proceedings of the International Conference on Advanced Intelligent Systems and Informatics*, Cairo, Egypt, Oct. 26-28, 2019. to be published.
- [4] Reshmi Krishnan, Anil. A. R., "A Survey on Image Matching Methods", International Journal of Latest Research in Engineering and Technology (IJLRET), vol. 2, no. 1, pp. 58-61, Jan., 2016. [Online].
- [5] Niangang Jiao, Wenchao Kang, Yuming Xianga, Hongjian You, "A Novel and Fast Corner Detection Method For SAR Imagery", International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences, vol. XLII, no. 2, Sep., 2017. DOI: 10.5194/isprs-archives-XLII-2-W7-605-2017

- [6] A.A. Karim, E. F. Nasser, "Improvement of Corner Detection Algorithms (Harris, FAST and SUSAN) Based on Reduction of Features Space and Complexity Time", *Engineering and Technology Journal*, vol. 35, no. B2, pp. 112-118, Dec., 2017.
- [7] Ehab Salahat, Murad Qasaimeh, "Recent Advances in Features Extraction and Description Algorithms: A Comprehensive Survey", Annual IEEE Industrial Electronics Societys 18th International Conf. on Industrial Technology (ICIT), pp. 22-25, March 22-25, 2017. arXiv:1703.06376 [cs.CV].
- [8] M. Hassaballah and Ali Ismail Awad, "Image Feature Detectors and Descriptors: Foundations and Applications", Studies in Computational Intelligence, vol.630, Springer Nature, Switzerland, 2016, pp. 11-45.
- [9] Arthur Ardeshir Goshtasby, "Theory and Applications of Image Registration", 1st ed., JohnWiley & Sons, Inc, 2017.
- [10] Tinne Tuytelaars, Krystian Mikolajczyk, "Local Invariant Feature Detector: A Survey", Foundations and trends in Computer Graphics and Vision, vol. 3, no. 3, pp. 177-280, 2008, DOI: 10.1561/0600000017.
- [11] E.R. Davies, "Computer Vision Principles, Algorithms Applications, Learning", 5th ed., Elsevier Inc., 2018.
- [12] K Y Kok and P Rajendran, "Validation of Harris Detector and Eigen Features Detector", International Conference on Aerospace and Mechanical Engineering (AeroMech17), Penang, Malaysia, Nov. 21-22, 2017, IOP Conference Series: Materials Science and Engineering, vol. 370, 2018. DOI:10.1088/1757-899X/370/1/012013.
- [13] Javier Sanchez, Nelson Monzon, Agustin Salgado, "An Analysis and Implementation of the Harris Corner Detector", *Image Processing On Line*, vol. 8, pp: 305-328, 2018. DOI: 10.5201/ipol.2018.229. [Online].
- [14] Harris, C. and Stephens, M., "A Combined Corner and Edge Detector", Proceedings of the 4<sup>th</sup> Alvey Vision Conference, pp. 147-151, Manchester, Aug. 31-Sep. 2, 1988.
- [15] H. P. Moravec, "Toward Automatic Visual Obstacle Avoidance," Proc. 5<sup>th</sup> of International Joint Conference on Artificial Intelligence (IJCAI), vol. 1, pp.584, Cambridge, MA, Aug. 22-25, 1977.
- [16] Shi, J., and C. Tomasi., "Good Features to Track", Proceedings of IEEE Conference on Computer Vision and Pattern Recognition (CVPR94), Seattle, USA, June 21-23, 1994, DOI: 10.1109/CVPR.1994.323794.
- [17] Peter Kovesi, "Image Features From Phase Congruency", Technical report 95/4, Department of Computer Science, The University of Western Australia, Australia, March, 1995.
- [18] Peter Kovesi, "Image Features From Phase Congruency", Journal of Computer Vision Research (Videre), vol. 1, no. 3, MIT Press Journals 1999
- [19] Peter Kovesi, "Phase congruency: A low-level image invariant", An International Journal of Perception, Attention, Memory, and Action (Psychological Research), vol. 64, pp:136-148, Springer Nature Switzerland, Dec., 2000.
- [20] Peter Kovesi, "Phase Congruency Detects Corners and Edges", Proceeding of 7<sup>th</sup> Digital Image Computing: Techniques and Applications(DICTA), pp. 309-318, Macquarie University, Sydney, Australia, Dec. 10-12, 2003.
- [21] Wenping Ma, Yue Wu, Shaodi Liu, Qingxiu Su, and Yong Zhong, "Remote Sensing Image Registration Based on Phase Congruency Feature Detection and Spatial Constraint Matching", *IEEE Access*, vol. 6, pp. 77554-77567, Dec., 2018. DOI: 10.1109/ACCESS.2018.2883410.
- [22] Zheng Liu and Robert Laganiere, "On The Use of Phase Congruency to Evaluate Image Similarity ", IEEE International Conference on Acoustics Speech and Signal Processing Proceedings, Toulouse, France, May 14-19, 2006, DOI: 10.1109/ICASSP.2006.
- [23] Adrian Burlacu and Corneliu Lazer, "Image Features Detection using Phase Congruency and Its Application in Visual Servoing", 4th International Conference on Intelligent Computer Communication and Processing, Cluj-Napoca, Romania, pp. 47-52, Aug. 28-30, 2008, DOI: 10.1109/ICCP.2008.4648353.
- [24] Zaafouri Ahmed, Mounir Sayadi and Farhat Faniech, "Satellite Images features Extraction using Phase Congruency model", *International Journal of Computer Science and Network Security (IJCSNS)*, vol. 9, no. 2, pp. 192-197, Feb., 2009.
- [25] Jyoti Malik, G. Sainarayanan and Ratna Dahiya, "Corner Detection Using Phase Congruency Features", International Conference on Signal and Image Processing, pp: 217-221, Chennai, India, Dec. 15-17, 2010, DOI: 10.1109/ICSIP.2010.5697472.

- [26] Y. L. Malathi Latha, "Local Feature Integration Method Using Phase Congruency for Palm Print Authentication", *International Journal of Image and Graphics (IJIG)*, vol. 15, no. 3, pp. 15500081-155000817, 2015, DOI: 10.1142/S0219467815500084.
- [27] Qiang Zhang, Yabin Wang, Long Wang, "Registration of images with affine geometric distortion based on maximally stable extremal regions and phase congruency", *Image and Vision Computing*, vol. 36, pp. 23-39, ELSEVIER, April, 2015, DOI: 10.1016/j.imavis.2015.01.008
- [28] A.F. Cinar, S.M. Barhli, D. Hollis, M. Flansbjer, R.A. Tomlinson, T.J. Marrow, M. Mostafavi, "An autonomous surface discontinuity detection and quantification method by digital image correlation and phase congruency", *Journal of Optics and Lasers in Engineering*, vol. 96, pp. 94-106, ELSEVIER, 2017. DOI: 10.1016/j.optlaseng.2017.04.010.
- [29] Clive Trenton, Andrew Lambert, "Detecting Space Debris using Phase Congruency", Engineering Project Report, University of New South Wales at Australian (UNSW), Defense Force Academy (DFA), Australian, Oct. 23, 2017.
- [30] Shoba Dyre, C.P. Sumathi, "Enhanced Gray Scale Skeletonization of Fingerprint Ridges Using Parallel Algorithm", Recent Trends in Image Processing and Pattern Recognition, vol. 709, Springer, 2017. DOI: 10.1007/978-981-10-4859-3\_39.
- [31] Yuming Xiang, Feng Wang, Ling Wan and Hongjian You, "SAR-PC: Edge Detection in SAR Images via an Advanced Phase Congruency Model", Remote Sensing - Multidisciplinary Open Access Journal (MDPI), vol. 9, no. 3 Feb., 2017, DOI: 10.3390/rs9030209.
- [32] Yuanxin Ye, Jie Shan, Siyuan Hao, Lorenzo Bruzzone, Yao Qin, "A local phase based invariant feature for remote sensing image matching", *Journal of Photogrammetry and Remote Sensing (ISPRS)*, vol. 142, pp. 205-221, June, 2018, DOI: 10.1016/j.isprsjprs.2018.06.010.
- [33] Zhen Ye, Xiaohua Tong et al., "Illumination-Robust Subpixel Fourier-Based Image Correlation Methods Based on Phase Congruency", *IEEE Transaction on Geoscience and Remote Sensing*, vol. 57, no. 4, pp. 1995-2008, April, 2019, DOI: 10.1109/TGRS.2018.2870422.
- [34] Joanna Janicka, Jacek Rapinski, "Outliers Detection By RANSAC Algorithm In The TransformationOf 2D Coordinate Frames", *Bulletin of Geodetic Sciences Journal*, vol. 20, no. 3, pp. 610-625, Sep., 2014, DOI: 10.1590/S1982-21702014000300035.
- [35] Gaetano Castaldo, Antonio Angrisano, Salvatore Gaglione, Salvatore Troisi, "P-RANSAC: An Integrity Monitoring Approach for GNSS Signal Degraded Scenario", *International Journal of Navigation and Observation*, vol. 2014, Sep., 2014, DOI: 10.1155/2014/173818.
- [36] Serkan Dusmez, Mehrdad Heydarzadeh, Mehrdad Nourani, and Bilal Akin, "Remaining Useful Lifetime Estimation for Power MOSFETs Under Thermal Stress With RANSAC Outlier Removal", *IEEE Transaction On Industrial Informatics*, vol. 13, no. 3, pp. 1271-1279, June, 2017, DOI: 10.1109/TII.2017.2665668.
- [37] Rosten, E., Drummond, T., "Machine learning for high speed corner detection", 9<sup>th</sup> European Conference on Computer Vision (ECCV2006), *Lecture Notes in Computer Science*, vol. 3951, Part I, pp: 430-443, Springer, Berlin, Heidelberg, 2006, DOI: 10.1007/11744023\_3.